Nonparametric estimation of English auctions with selective entry: An application to online judicial auctions

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Abstract

This paper proposes an estimation approach following the constructive identification strategy of Athey and Haile (2002) and Gentry and Li (2014) with adaption in the context of ascending auctions with selective entry. Our estimators are shown to be consistent in a large sample and to perform well in a finite sample by a simulation study. We apply our estimation approach to the Alibaba online judicial auctions of used cars to recover the bounds of conditional value distribution and the entry cost. The bounds estimates of both conditional value distribution and entry cost are quite tight (resp. relatively wide) for middle-valued (resp. low-valued or high-valued) signal, and the CDFs of conditional value distribution given signal comply with the law of ordered dominance. Finally, our counterfactual analysis indicates that (i) the ascending auction yields a higher revenue than the first-price sealed bid auction; and (ii) the revenue can be improved significantly when the entry cost is cut by half.

Keywords: online judicial auction, selective entry, bound estimation, counterfactual analysis
JEL Classification: C13, C14, C15, C16

1 Introduction

There has been a very rapid development of judicial auctions in the Chinese market during the past decade. A total amount of around 190 billion Chinese yuan (CNY) has been transacted by online judicial auctions during all years before 2017, but the transaction amount dramatically increased to around 400 billion CNY in a single year of 2020. With such a high growth, the total transaction amount of Chinese online judicial auctions has exceeded 2 trillion CNY over all years until September 2022 (The Supreme People's Court of China, 2022). Also, the number of attendants and auctioned objects has also increased considerably during the same period. For example, in the year of 2020, about 390 thousand people attended the online judicial auctions of more than 573 thousand auctioned objects. Online judicial auction has gradually become an increasingly popular auction format, and has produced huge amount of bids data available for researchers.

Endogenous participation is one key feature of online judicial auctions in the Chinese market. For example, the average participation rate is 75.32% in the judicial auctions of used cars on the Alibaba

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platform during March to October 2020. Endogenous participation has also been documented in other auction datasets. See, for example, Bajari and Hortaçsu (2003); Hendricks et al. (2003); Li and Zheng (2009); Li and Zhang (2010); Athey et al. (2011); Krasnokutskaya and Seim (2011), among others. To accommodate endogenous participation in auctions, Marmer et al. (2013) develop a so-called Affiliated-Signal (AS) model (drawing structure from Ye (2007)) which nests both the Levin and Smith (1994) model and the Samuelson (1985) model depending on how informative the entry signal is about the value of a bidder in an auction. Its identification is extensively studied by Gentry and Li (2014). Specifically, they exploit the variation in the number of potential bidders and some auction-level instruments to construct bounds on conditional value distribution and entry cost. In particular, point identification can be achieved in the case of continuous entry variation. For a recent survey of the literature on auctions with selective entry, see, e.g., Gentry et al. (2018); Perrigne and Vuong (2021), among others.

This paper investigates Chinese online judicial auctions by applying our new estimation approach (on ascending auctions with selective entry) to a unique dataset of judicial auctions of used cars held on Alibaba. Our estimation approach is proposed by following the constructive identification strategy of Athey and Haile (2002) and Gentry and Li (2014) (with adaption) in the context of ascending auctions with selective entry. We also establish the consistency of our estimators in a large sample. Specifically, in the post-entry stage, we estimate the post-entry value distribution using the identification strategy of Athey and Haile (2002); in the entry stage, our estimation then follows the partial identification of Gentry and Li (2014) in the case without any auction level instrument variable.

Our estimation method is then applied to the alibaba judicial auctions of used cars to recover the bounds of conditional value distribution and the entry cost. Our bound estimates of conditional value distribution is quite tight for a middle-valued signal. In our counterfactual analysis, we find that (i) the ascending auction yields a higher revenue than the first-price sealed bid auction; and (ii) The revenue can be improved significantly when the entry cost is cut by half.

Our paper first contributes to the increasing empirical literature using auction datasets in the Chinese market. Cai et al. (2013) studied land auctions in China and found evidence of corruption by choosing the auction format. Gao et al. (2018) studied initial public offering (IPO) auctions in China and analyzed the bidding behavior of institutional investors. Luo et al. (2021) introduced an order statistics approach and applied their method to Chinese online judicial auctions of residential property. Liu and Xu (2022) adapted Haile and Tamer (2003)'s estimation approach to address the issue of bound crossing without tuning parameter in an incomplete model of English auction without entry. There seems to be no straightforward way to incorporate selective entry into the incomplete model of English auctions considered in Haile and Tamer (2003). In English auctions with selective entry, this article estimates the value distribution and entry cost bounds following the constructive identification strategy of Athey and Haile (2002) and Gentry and Li (2014) with some adaptations. Also, we establish the consistency of our estimators in a large sample.

We also contribute to the literature of auctions with selective entry.[1] Selective entry breaks the exogenous participation assumption, which is a key restriction to identify various auction models with monotone strategies. See, e.g., Guerre et al. (2000, 2009); Li and Liu (2022), for such identifica-

[1]Selective entry introduces some form of non-random sample selection through censoring. Sample selection model with censored selection has been studied, e.g., by Lee (1994); Chen (1997) in a semiparametric mean regression, and, e.g., Chen et al. (2023a,b) in a semiparametric quantile/distribution regression.

tion results; and Liu and Luo (2017); Liu and Vuong (2021) for testing the restrictions of exogenous participation and monotone strategies in auctions.[2] Under selective entry, Li (2005) introduced an MSM estimator to estimate the value distribution parametrically using the known bids and number of bidders. Marmer et al. (2013) proposed a general framework of the AS model to characterize selective entry in auctions and designed a non-parametric test to discriminate its two polar cases given by Levin and Smith (1994) and Samuelson (1985), respectively, from the general model. Chen et al. (2020) identified and estimated the AS model with risk aversion in the context of first-price auctions. Relative to Roberts and Sweeting (2010) who estimate a parametric version of ascending auctions with selective entry, our paper considers non-parametric estimation of bounds of value distribution and entry cost in an English auction with entry. We also apply our approach to an application of Alibaba online auctions which is different from Roberts and Sweeting (2010) who investigate the USFS timber auctions.

We organize the rest of this paper as follows. A unique dataset is described and the facts in reduced form are shown in section 2. We introduce the econometric methodology used in our application in Section 3. Section 4 provides a simulation study to show the finite sample performance of our estimators. The main empirical results are shown in Section 5. Section 6 concludes our paper. An appendix collects the proofs for all theorems and lemmas.

2 Data

2.1 Data description

We first briefly introduce the mechanism of Alibaba judicial auctions of used cars. Before entering an auction, each bidder can observe the brand of the car, the number of potential bidders, the starting price, and the appraisal value. Additionally, a photo of the used car is also available. Each potential bidder receives a private signal about her own value. After paying an entry cost, a potential bidder enters the auction and becomes an active bidder. The entry cost accounts for the deposit, time cost, cost of studying the auction web page, and cost of observing the cars on the spot (if happens). After entering the auction, active bidders obtain all information about the object and realize their own valuations. Only bidders with values higher than the start price make bids. After entry, the auction takes the format of first price ascending auction. We assume that the bidders are risk-neutral and have symmetric independent private values.

A dataset of used cars' judicial auctions was collected from the Alibaba judicial auction platform from March to October in the year 2020. Many auction-level characteristics are available on the platform. The traveling distance and the appraisal value of the auctioned car are given. We also collect the official prices of new cars from a website called "Youjia", which is owned by the tech giant Baidu.[3] The official price of a new car is used to measure the value of the brand of a used car with the same brand. We collect the number of individuals who sign up for the auction as the number of potential bidders. In addition, the following information is also available for each auction: transaction price, start price, deposit for participation, number of photos, number of applicants, number of bidders, and number of bid rounds.

[2]Monotonicity restrictions have been used widely to identify various economic models, including binary games (see, e.g., Liu et al., 2017; Liu and Xu, 2016), and generalized additive models (see, e.g., Chen et al., 2023c).

[3]https://www.yoojia.com/

Table 2.1: Summary statistics

	Mean	Std. Dev	Min	Max
Transaction price	77.211	60.939	1.57	277.00
Official price	210.046	173.635	28.9	913
Traveling distance	111.628	76.506	0.307	378.903
appraisal value	81.312	66.804	1	340
Start price	63.370	52.530	0.216	250
Deposit	9.358	8.478	0.2	50
Bid increment	0.730	0.883	0.01	20
Number of photos	6.163	1.891	1	15
Number of bidders	4.538	1.801	2	9
Number of potential bidders	5.778	2.133	2	9
Number of bid rounds	17.275	9.616	2	45

Notes: all prices, deposit, and bid increment are in thousand CNY; and the traveling distance is in thousand kilometers.

We dropped criminal judicial auctions because the characteristics of criminal cases are not available on the auction platform. In addition, we exclude data with only one bidder or with missing information in the characteristics of interest. We further drop the auctions with more than 9 potential bidders because the number of those auctions is too small. Furthermore, auctions with the highest 2% travel distance or with the highest 2% ratio from start price to appraisal are excluded. Eventually, 808 auctions are included, in which both the number of potential bidders and actual bidders range from 2 to 9. Table 2.1 provides the summary statistic of our data. All prices, deposit, and bid increment are measured in thousand CNY and the traveling distance is in thousand kilometers.

2.2 Reduced-form facts

Following Liu and Xu (2022), we estimate a linear regression model to assess the effects of interfering factors on the transaction price in this section. We regress the logarithm of transaction price on variables that reflect the characteristics of the auctions and the objects, which are the logarithm of official price, traveling distance, start price, deposit, and the number of potential bidders. Table 2.2 presents the results of the linear regression.

The above results draw a rough picture of the used cars' data. The outcome of the linear regression model indicates that the official price and the starting price, the deposit value, and the number of potential bidders of the auction have a positive effect on the transaction price of the object, and the effects are all significant at a significance level of 1%. As shown in Table 2.2, fixing all other factors, 1% increase in official price (or deposit) will raise the transaction price by 0.106% (or 0.276%). While traveling distance has an insignificantly negative effect on transaction price at a significance level of 5%. The increase in 1% traveling distance will decrease the transaction price by 0.015%. This result is intuitive since more traveling distances reflect the decline of car's performance, thus reducing its

Table 2.2: Linear regression result

Dependent Variable: log (transaction price)

Regressor	Coefficient estimate	t value
log (appraisal value)	0.566 ***	25.65
log (official price)	0.106 ***	5.16
log (traveling distance)	-0.015	-1.24
log (deposit)	0.276 ***	12.71
number of potential bidders	0.039 ***	8.94
Cons	0.527 ***	6.03

Notes: (i) all prices and deposits are in thousand CNY; and the traveling distance is in thousand kilometers.

(ii) *** stands for significance at a level of 1%.

transaction price.

In reduced-form analysis, the endogeneity issue can be addressed by some auction-level instrumental variables. However, the reduced-form approach cannot conduct counterfactual analysis in an alternative auction format or a different entry cost level. Therefore, we propose a structural approach that takes advantage of the variation of the number of potential bidders to obtain the bounds of the underlying conditional value distribution and entry cost in the next section.

3 Econometric methodology

We follow Athey and Haile (2002) and Gentry and Li (2014) to non-parametrically identify the bounds of bidder's conditional value distribution given the signal and the bounds of the entry cost. The strategy of Athey and Haile (2002) is used to identify the post-entry value distribution that is required by the identification strategy of Gentry and Li (2014). We briefly describe the model, the identification strategy, and our proposed estimation procedure in this section. For simplicity of discussion, we abbreviate the auction-level characteristics in this section. We denote random variables in the upper case, while their realizations are in the lower case.

3.1 Model

Let $N \in \{2, \ldots, \overline{N}\}$. N potential bidders participate in a first price ascending auction with independent and identical value distribution $F(\cdot)$. Before entry, both their own values V_i and the others' V_{-i} are not known, while they all receive a signal S_i of their values. V_i and S_i are drawn from the same joint distribution $F(v, s)$, and the marginal distribution of the signal S_i is normalized to be uniform on [0,1]. We follow the AS model in Gentry and Li (2014) and assume that the signal S_i is affiliated with V_i. After receiving the signal, the potential bidder will decide whether he will enter this auction by paying an entry cost c. The threshold signal level s_N^* for a potential bidder is identified by the indifference between entering and not entering when all other potential bidders receive the same signal level given a specific value of N, namely $\Pi(s_N^*; s_N^*, N) = c$, where $\Pi(s; \tilde{s}, N)$ represents the expected profit of a potential bidder with signal s and the signals of other competitors are $\tilde{s}$ given the value of N. The seller sets a reserve price r. After paying the entry cost, n active bidders whose values are higher than r bid in an ascending outcry mechanism, and everyone can bid one or more times. The highest bidder will win this auction and pay his own bid to obtain the object. Denote the post-entry value distribution with N potential bidders as $F^*(\cdot, s_N^*)$, which can be written as $F^*(v, s_N^*) = \Pr(V \leq v | S \geq s_N^*)$ by definition.

3.2 Identification

We first give our identification assumptions as follows.

Assumption A. (i) V_i, S_i are drawn from a joint distribution symmetrically and it satisties:

1. V_i is bounded by $\mathcal{V} = [\underline{v}, \overline{v}]$, and the joint distribution $F(v, s)$ is continuous in (v, s).
2. If $s' \geq s$, then $F(v|s') \leq F(v|s)$.
3. For any $j \neq i$, $(V_i, S_i) \perp (V_j, S_j)$.
4. We normalize the signals S_i to be uniformly and marginally distributed on $[0, 1]$, that is, $S_i \sim U[0, 1]$.

(ii) Conditional expectation $E[V_i | S_i = s]$ is continuous in s on $[0, 1]$.

(iii) $F(v, s|N) = F(v, s)$ and $c(N) = c$ hold for any $N \in \mathcal{N}$.

Parts (i)-(iii) of our Assumption A correspond to Assumptions 1, 2, and 4 of Gentry and Li (2014). Note that Assumption 3 of Gentry and Li (2014) is satisfied by our second-stage mechanism of ascending auctions, and their Assumption 5 holds by applying the identification strategy of Athey and Haile (2002) to the stage 2 mechanism.

Next, we present our identification strategy. First, the post-entry value distribution is nonparametrically identified by the distribution of winning bids following the order statistic identification strategy proposed by Athey and Haile (2002), since the equilibrium winning bid is equal to the second highest value of all active bidders if there are at least two entrants. Second, under Assumption A, the conditional value distribution $F(v|\hat{s})$ (given signal s being the cutoff $\hat{s}$) and the entry cost c are partially identified from the post-entry value distribution by Gentry and Li (2014)[4].

We then present two monotonicity results on the threshold signal and the post-entry value distribution in our auction model.

Lemma 1. *Under Assumption A, if $N' > N$, then we have (i) $s_{N'}^* \geq s_N^*$ and (ii) $F^*(v, s_{N'}^*) \leq F^*(v, s_N^*)$ for each v.*

Part (i) of Lemma 1 states that the threshold signal increases in the number of potential bidders N. It is the last point of Proposition 1 of Gentry and Li (2014). Part (ii) further indicates that the post-entry value distribution also shows stochastic dominance over the number of potential bidders N, that is, the post-entry value distribution with a bigger N stochastically dominates that with a smaller N.

[4]In contrast to the second price sealed bid auctions where all bidders submit their true values, not every bidder can submit the value as her highest bid in our English auction model due to the constraint in bidding order. However, the bidder with the highest value will win the last round of competition against the bidder with the second highest value simply by bidding the second highest value. In that sense, only the transaction price reveals the second-highest value in equilibrium. We use the strategy of Athey and Haile (2002) to identify the value distribution of the entrants from the distribution of the transaction price, and then apply the strategy of Gentry and Li (2014) to further identify the bounds of the value distribution of potential entrants, as well as the bounds of entry cost.

This stochastic dominance is an implication of Assumption A (i.2) and Proposition 1 of Gentry and Li (2014).

3.3 Estimation

In this paper, we propose an estimation approach in three steps. In the first step, we estimate the threshold signal $\hat{s}_N$ using a GMM type approach. In the second step, we estimate the post-entry value distribution $\hat{F}^*(v, \hat{s}_N)$ following the identification strategy of Athey and Haile (2002). In the third step, we estimate the bounds of the conditional value distribution given the signal and the entry cost following the partial identification strategy of Gentry and Li (2014) when there is no instrumental variable at the auction level Z that affects the entry cost but not the value distribution.

In the first step, the threshold signal $\hat{s}_N$ is estimated using a GMM method. For auctions with N potential bidders, $n = 1, \ldots, \overline{n}$ ($\overline{n} = N$) active bidders may enter and bid.[5] The number of actual bidders n is not observed before the bidders bid. Let T_n with $n = 1, \ldots, \overline{n}$ denote the number of auctions with n actual bidders. We obtain the estimate of threshold signal $\hat{s}_N$ under N as follows:

$$\begin{aligned}\hat{s}_N = argmin_{s_N}[(T_{\overline{n}} - &\frac{T_2}{C_N^2 s_N^{(N-2)}(1-s_N)^2} \cdot (1-s_N)^N)^2 \cdot 1\{N \geq 5\} \\ &+ (T_{\overline{n}-1} - \frac{T_2}{C_N^2 s_N (1-s_N)^2} \cdot s_N (1-s_N)^{N-1})^2 \cdot 1\{N \geq 4\} \\ &+ \cdots + (T_3 - \frac{T_2}{C_N^2 s_N (1-s_N)^2} \cdot s_N^{N-3}(1-s_N)^3)^2]\end{aligned} \tag{1}$$

In the second step, we utilize the identification strategy of Athey and Haile (2002) to obtain an estimate of the post-entry value distribution $\hat{F}^*(v, \hat{s}_N)$ for a fixed $N \in \{2, \ldots, \overline{N}\}$. In cases with at least two entrants[6], the equilibrium condition implies that $\hat{F}^*_{n|N}(v, \hat{s}_N) = \phi_n(\hat{G}_{n|N}(v))$, where $\hat{G}_{n|N}(v)$ is the estimate of the winning bid distribution with N potential bidders and n actual bidders, and the function $\phi_n(\cdot)$ is implicitly defined by $x = n \cdot \phi_n(x)^{n-1} - (n-1) \cdot \phi_n(x)^n$. Next, we obtain an estimate of the post-entry value distribution with N potential bidders by averaging all $\hat{F}^*_{n|N}(v, \hat{s}_N)$ over $n = 1, \ldots, N$ as follows:

$$\hat{F}^*(v, \hat{s}_N) = \sum_{n=1}^{N} \hat{F}^*_{n|N}(v, \hat{s}_N) \cdot \frac{A^2_{n|N}}{\sum_{n=1}^{N} A^2_{n|N}} \tag{2}$$

where $A_{n|N}$ represents the number of auctions with n entrants from N potential bidders. Here, a weight of $w_{n|N} = \frac{A^2_{n|N}}{\sum_{n=1}^{N} A^2_{n|N}}$ is employed.[7].

In practice, the post entry value distribution CDFs with a different number of potential bidders N can cross at some values of v due to the estimation error in a finite sample. This violates the stochastic dominance order implied by Lemma 1 which indicates that the CDF of the post-entry value distribution with a bigger N is always below the one with a smaller N in any v. In this case, we can

[5] The case of $n = 0$ corresponds to a failed transaction.

[6] If there is only one active bidder, she will bid the starting value in equilibrium. But auctions with only one active bidder are not included in the sample for our estimation.

[7] Our choice of weights puts more weights on the estimate of $\hat{F}^*_{n|N}(v, \hat{s}_N)$ with large value of $A_{n|N}$ than the traditional weights of simply number of active bidders (instead of its square). But the empirical results do not change significantly if traditional weights were used.

adjust the post-entry value distribution estimators so that they satisfy such a stochastic dominance order over N. One way is to adjust them as $\tilde{F}^*(v,\hat{s}_N) = \min_{N'=2,\dots,N} \hat{F}^*(v,\hat{s}_{N'})$ for all $N = 2,\dots,\overline{N}$. The second way is to adjust them as $\tilde{F}^*(v,\hat{s}_N) = \max_{N'=N,\dots,\overline{N}} \hat{F}^*(v,\hat{s}_{N'})$ for all $N = 2,\dots,\overline{N}$. A third way is to mix the former two ways to keep the stochastic dominance order on N. In particular, we can also implement a smoothing method of Haile and Tamer (2003) with a tuning parameter $h_N = \pm\sqrt{T_N}$ to approximate the maximum and minimum operators in the above three ways[8].

In the third step, with the nonparametric estimates of threshold signal $\hat{s}_N$ and post-entry value distribution $\hat{F}^*(v,\hat{s}_N)$, we can firstly obtain the estimates of bounds on conditional value distribution (given signal $\hat{s} \in [0,1]$) $\hat{F}^+(v|\hat{s})$,$\hat{F}^-(v|\hat{s})$ as follows:

$$\hat{F}^+(v|\hat{s}) = \begin{cases} \tilde{\tilde{F}}^+(v|\hat{s}), & if \;\; \hat{s} \in \mathcal{S}, \\ \tilde{\tilde{F}}^+(v|t^-(\hat{s})), & if \;\; \hat{s} \notin \mathcal{S}, \end{cases} \tag{3}$$

$$\hat{F}^-(v|\hat{s}) = \begin{cases} \tilde{\tilde{F}}^-(v|\hat{s}), & if \;\; \hat{s} \in \mathcal{S}, \\ \tilde{\tilde{F}}^-(v|t^+(\hat{s})), & if \;\; \hat{s} \notin \mathcal{S}. \end{cases} \tag{4}$$

where

$$\tilde{\tilde{F}}^+(v|\hat{s}) = \begin{cases} \lim_{t\uparrow t^-(\hat{s})} \left\{ \frac{(1-t)\hat{F}^*(v;t)-(1-\hat{s})\hat{F}^*(v;\hat{s})}{\hat{s}-t} \right\}, \\ if \;\; \hat{s} \in \mathcal{S} \;\; and \;\; t^-(\hat{s}) \in \mathcal{S}, \\ 1, \quad otherwise, \end{cases}$$

$$\tilde{\tilde{F}}^-(v|\hat{s}) = \begin{cases} \lim_{t\uparrow t^+(\hat{s})} \left\{ \frac{(1-\hat{s})\hat{F}^*(v;\hat{s})-(1-t)\hat{F}^*(v;t)}{t-\hat{s}} \right\}, if \;\; \hat{s} \in \mathcal{S}, \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad otherwise. \end{cases}$$

and

$$t^+(\hat{s}) = \begin{cases} inf\,\{t \in \mathcal{S} | t > \hat{s}\}, if \sup\{\mathcal{S}\} > \hat{s}, \\ 1, \qquad\qquad\qquad\qquad otherwise, \end{cases}$$

$$t^-(\hat{s}) = \begin{cases} inf\,\{t \in \mathcal{S} | t < \hat{s}\}, if \sup\{\mathcal{S}\} < \hat{s}, \\ 0, \qquad\qquad\qquad\qquad otherwise. \end{cases}$$

Secondly, we can estimate the bounds on entry cost $\hat{c}^+$ and $\hat{c}^-$ as follows[9]:

$$\hat{c}^+ = \frac{1}{J}\sum_{j=1}^{J} \hat{c}_N^+, \tag{5}$$

$$\hat{c}^- = \frac{1}{J}\sum_{j=1}^{J} \hat{c}_N^-. \tag{6}$$

[8]About our adaption in estimation, we mainly follow the identification strategies of Athey and Haile (2002) and Gentry and Li (2014) to construct our estimators with two exceptions. The first exception is to get an initial estimator of the post-entry value distribution $\hat{F}^*(v,\hat{s}_N)$ by averaging all $\hat{F}_{n|N}(v,\hat{s}_N)$, which is estimated by following Athey and Haile (2002)'s identification strategy over $n = 1,\dots,N$ for a given number N of potential bidders. The second exception is to obtain the final estimator $\tilde{F}^*(v,\hat{s}_N)$ of the post-entry value distribution by further adjusting the initial estimator $\hat{F}^*(v,\hat{s}_N)$ to satisfy the stochastic dominance properties over N.

[9]Gentry and Li (2014) do not deal with the reserve price directly. We do not deal with the reserve price in our estimation and empirical application, either.

where J is the number of N, and

$$\hat{c}_N^+ = \int_0^{\overline{v}} [1 - \hat{F}^-(y|\hat{s}_N)] \cdot [\hat{s}_N + (1-\hat{s}_N)\hat{F}^*(y;\hat{s}_N)]^{N-1} dy,$$
$$\hat{c}_N^- = \int_0^{\overline{v}} [1 - \hat{F}^+(y|\hat{s}_N)] \cdot [\hat{s}_N + (1-\hat{s}_N)\hat{F}^*(y;\hat{s}_N)]^{N-1} dy.$$

The consistency of our bound estimators is summarized by the following theorem.

Theorem 1. *Let T_N be the number of auctions with N potential bidders. Under Assumption A, when $\min_{N=2,\ldots,\overline{N}} T_N$ goes to infinity, we have the following.*

(i) For any $s \in [0,1]$, $\hat{F}^+(v|s)$, and $\hat{F}^-(v|s)$ respectively converge to their true distributions $F^+(v|s)$ and $F^-(v|s)$ for any v with probability of one.

(ii) $\hat{c}^+$ and $\hat{c}^-$, respectively, converge to their true values c^+ and c^- with probability of one.

4 A Simulation Study

We next conduct a Monte Carlo experiment to show the finite sample performance of our bound estimates of conditional value distributions and entry costs.

We adopt the same specification on the joint distribution of value V and signal S as the simulation study of Marmer et al. (2013). Specifically, we generate (Z_1, Z_2) by a bivariate normal distribution with means of zero and variances of one, and the correlation parameter ρ is between 0 and 1. The signal and value are generated by $S = \Phi(Z_1)$ and $V = \Phi(Z_2)$, respectively, where Φ is the standard normal CDF. Therefore, the conditional value distribution given signal S and the post entry value distribution are obtained as:

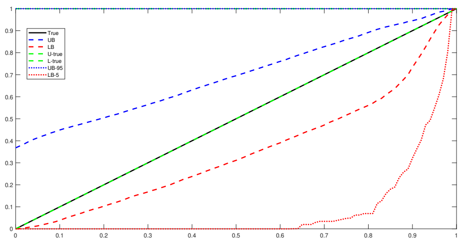
$$F(v|s) = P(V \le v|S) = P(Z_2 \le \Phi^{-1}(v)|\Phi^{-1}(S)) = \Phi(\frac{\Phi^{-1}(v) - \rho\Phi^{-1}(S)}{\sqrt{1-\rho^2}}) \tag{7}$$

$$F^*(v|N) = F(v|S \ge s^*(N)) = \frac{1}{1-s^*(N)} \int_{s^*(N)}^1 \Phi(\frac{\Phi^{-1}(v) - \rho\Phi^{-1}(s)}{\sqrt{1-\rho^2}}) ds \tag{8}$$

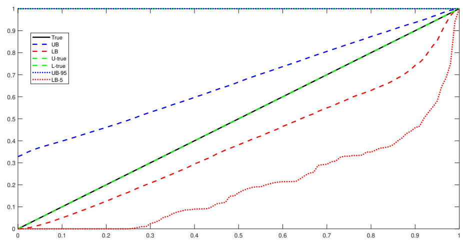
In each experiment, we set potential bidders $\mathcal{N} = \{2,3,4,5,6\}$, the numbers of auctions $L = 1000$ and 2000 , $\rho = 0, 0.25, 0.5$ and the entry cost $c = 0.17$. True signal thresholds under different ρ and N are shown in Table 4.1. In each auction, the value and signal of a potential bidder are generated by the above Gaussian copula with correlation parameter of ρ. Bidders with signals higher than the threshold will enter the auction, and their bids are generated by Example 2 in Appendix B of Haile and Tamer (2003). We set the bid increment $\Delta = 0.001$ and the probability of jump bidding $\lambda = 0$. Our simulation study has 500 repetitions of such an experiment.

Figures 1 - 3 show the mean estimates and 90% confidence bands of the bounds on the conditional value distribution given signal $S = s$ with various values of s and ρ and compare them with the true bounds of the conditional value distribution. Specifically, Figure 1 shows the results with different values of s and L given $\rho = 0$, Figure 2 shows the results with different values of s and L given $\rho = 0.25$, while Figure 3 shows the results given $\rho = 0.5$. They show that our bound estimators perform well in finite samples. Under 1000 auctions, the biases of our bound estimators are relatively small, and the 90% confidence band contains the true bounds, not far away from them. When the sample size increases to 2000, both the bias and confidence bands improve significantly. It should be noted that the estimated upper bounds are 1 for all values of $v \in [0, 1]$ in Figures 1 - 3 (a) and (b). The true value hence also locates between the upper and lower bounds. Another note is that the upper and lower bounds are equal to the true conditional value distribution when $\rho = 0$. In this case, s and v are uncorrelated, thus the post-entry distribution under different $\hat{s}_N$ is exactly the distribution of v. Therefore, the upper and lower bounds of the conditional value distribution are identical, and both are the distribution of v.

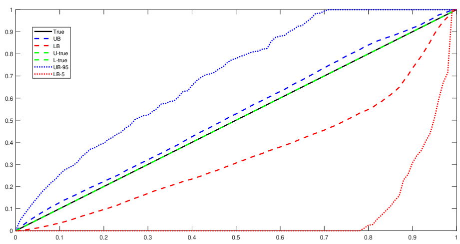
Table 4.2 show the estimated bounds on entry cost with various values of ρ. It presents the mean and standard deviation of estimated bounds on entry cost and compares them with the true bounds under different ρ. It shows that (i) the bound estimates of entry cost have relatively small bias and standard error under both $L = 1000$ and $L = 2000$, and (ii) they decline significantly when the sample size increases from $L = 1000$ to $L = 2000$.



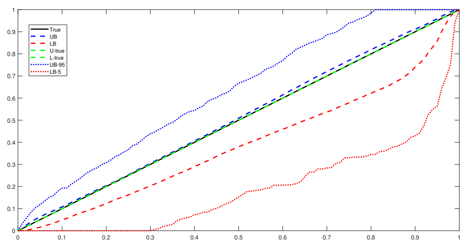
(a) s = 0.35, $\rho = 0$, L = 1000, replication = 500



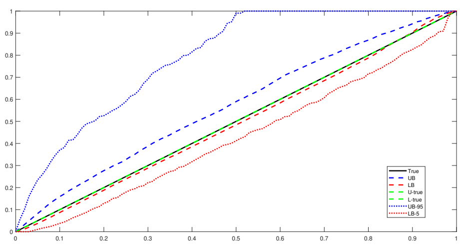
(b) s = 0.35, $\rho = 0$, L = 2000, replication = 500



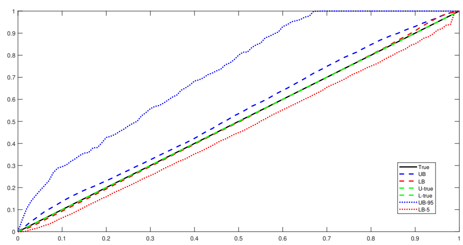
(c) s = 0.5, $\rho = 0$, L = 1000, replication = 500



(d) s = 0.5, $\rho = 0$, L = 2000, replication = 500



(e) s = 0.65, $\rho = 0$, L = 1000, replication = 500



(f) s = 0.65, $\rho = 0$, L = 2000, replication = 500

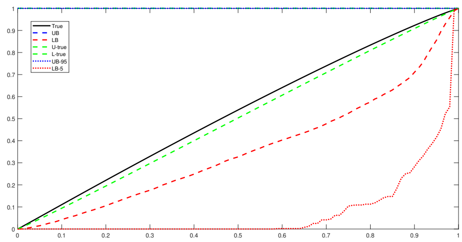
Figure 1: Mean and 90% confidence band of simulated conditional value distribution bounds - $\rho = 0$

Table 4.1: True signal threshold

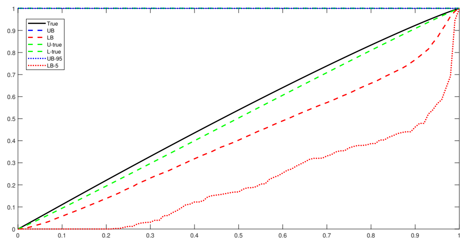
$\hat{s}_N$	N = 2	N = 3	N = 4	N = 5	N = 6
$\rho = 0$	0.010	0.343	0.518	0.621	0.688
$\rho = 0.25$	0.154	0.415	0.560	0.648	0.706
$\rho = 0.5$	0.250	0.468	0.592	0.670	0.723

Table 4.2: Estimated entry cost

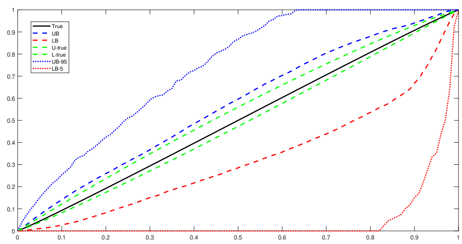
			$\rho = 0$	$\rho = 0.25$	$\rho = 0.5$
	True c		0.17	0.17	0.17
$L = 1000$	c^+	true	0.170	0.172	0.175
		mean	0.220	0.237	0.249
		std.	0.018	0.020	0.022
	c^-	true	0.170	0.168	0.167
		mean	0.120	0.114	0.114
		std.	0.014	0.015	0.014
$L = 2000$	c^+	true	0.170	0.172	0.175
		mean	0.203	0.218	0.228
		std.	0.013	0.015	0.015
	c^-	true	0.170	0.168	0.167
		mean	0.126	0.121	0.120
		std.	0.010	0.011	0.010
$L = 4000$	c^+	true	0.170	0.172	0.175
		mean	0.192	0.203	0.211
		std.	0.009	0.008	0.011
	c^-	true	0.170	0.168	0.167
		mean	0.132	0.127	0.125
		std.	0.008	0.008	0.008



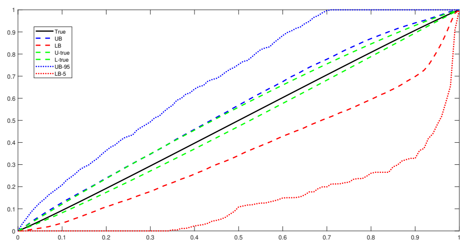
(a) s = 0.35, $\rho = 0.25$, L = 1000, replication = 500



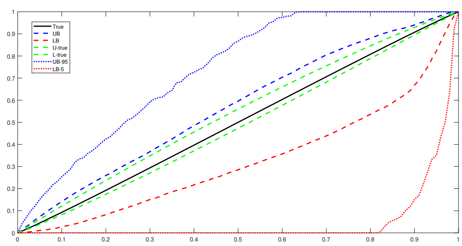
(b) s = 0.35, $\rho = 0.25$, L = 2000, replication = 500



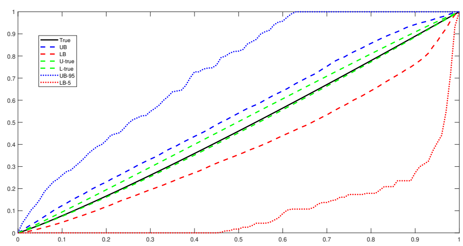
(c) s = 0.5, $\rho = 0.25$, L = 1000, replication = 500



(d) s = 0.5, $\rho = 0.25$, L = 2000, replication = 500

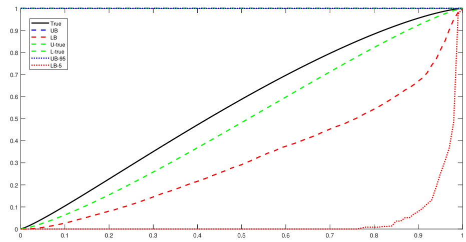


(e) s = 0.65, $\rho = 0.25$, L = 1000, replication = 500

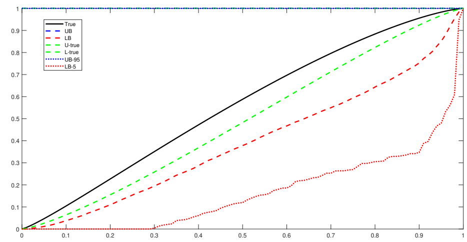


(f) s = 0.65, $\rho = 0.25$, L = 2000, replication = 500

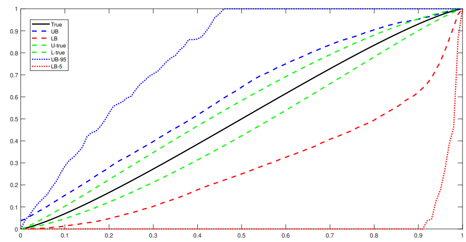
Figure 2: Mean and 90% confidence band of simulated conditional value distribution bounds - $\rho = 0.25$



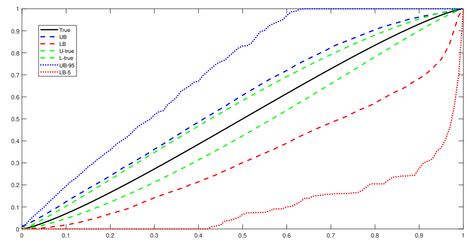
(a) s = 0.35, $\rho = 0.5$, L = 1000, replication = 500



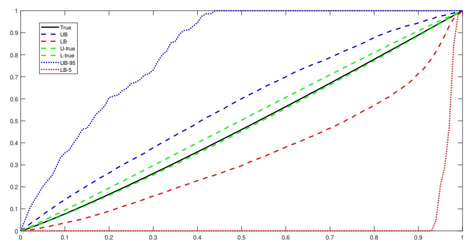
(b) s = 0.35, $\rho = 0.5$, L = 2000, replication = 500



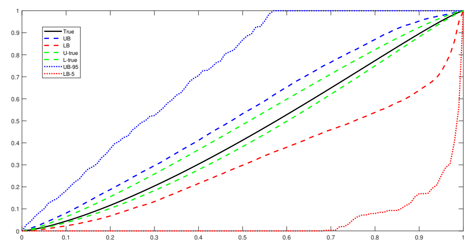
(c) s = 0.5, $\rho = 0.5$, L = 1000, replication = 500



(d) s = 0.5, $\rho = 0.5$, L = 2000, replication = 500



(e) s = 0.65, $\rho = 0.5$, L = 1000, replication = 500



(f) s = 0.65, $\rho = 0.5$, L = 2000, replication = 500

Figure 3: Mean and 90% confidence band of simulated conditional value distribution bounds - $\rho = 0.5$

Table 5.1: Estimated entry cost in application

	$\hat{c}^{+}$	std.	$\hat{c}^{-}$	std.
high appraisal	46.52	98.31	35.40	55.82
middle appraisal	18.94	20.06	12.11	3.48
low appraisal	8.51	4.42	5.19	1.97

5 Empirical results

We apply our estimation method to our data and obtain bound estimates of conditional value distribution as well as entry cost in this section. We divide the data into three groups by appraisal value to control the auction heterogeneity. After getting the conditional value distribution bound estimates, we estimate the bounds on entry cost of the three groups. We finally conduct two counterfactual analysis to explore the revenue change under different auction format or entry cost.

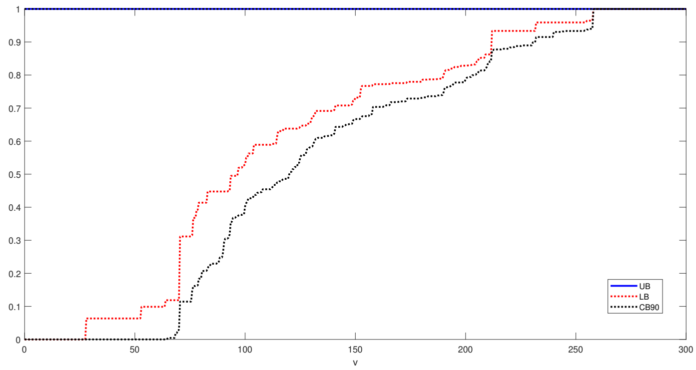
5.1 Bounds on conditional value distribution

We first estimate the bounds on conditional value distribution given the signal. Following Lu and Perrigne (2008), we choose the appraisal value to control the auction-level heterogeneity, since all other characteristics are considered when the used car for sale is appraised. Indeed, Table 2.2 also shows that the appraisal value coefficient has the highest estimate (and t-statistic value) among all characteristics in the reduced form analysis. We put all auctions with appraisal value below its 33th percentile (resp. between 33th and 67th percentiles, or above 67th percentile) in a group and call it a low (resp. middle or high) appraisal value group. For any s in [0,1], the bounds of the conditional value distribution are estimated by our method given in Section 3, and the results of $s = 0.35$, 0.5 and 0.65 are shown in Figures 4 to 6. 500 bootstrap repetitions are used to obtain the 90% confidence bands. The estimation results of the entry cost bounds are given by Table 5.1.

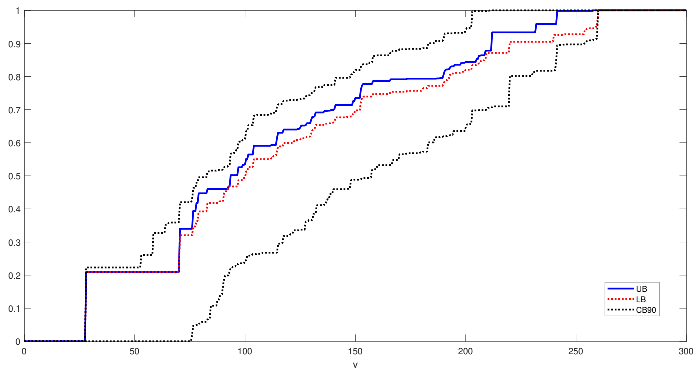
There are three interesting patterns in our estimate results. First, the bounds of the conditional value distribution are relatively tight for the signal s in the middle, while they are relatively wide for the signal s close to the boundaries. The upper bound is very close to (resp. relatively far away from) the lower bound when $s = 0.5$ (resp. when $s = 0.35$ and 0.65). Second, the bound estimates imply a stochastic dominance relation among the value distributions according to the appraisal value. Fix the value of the signal s, the conditional value distribution of the high appraisal group dominates the one of the middle appraisal group, and the latter dominates the one of the low appraisal group. Third, the bound estimates of entry cost also demonstrate a pattern of monotonicity over the appraisal value: the higher the appraisal value, the higher the entry-cost bound estimates are.

5.2 Counterfactual Analysis

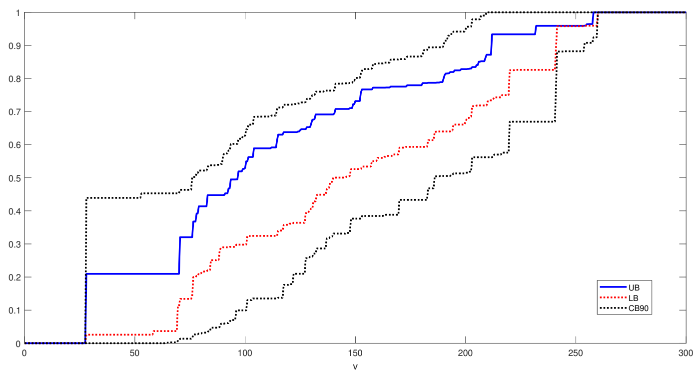
In this section, we propose two counterfactual analysis to explore the revenue difference by the change in auction format and entry cost. We first switch from ascending auction to first-price sealed bid auction and compare the winning bid under the two different auctions. Second, we cut and add the entry cost by half and obtain the bound estimates of revenue change.



(a) s = 0.35



(b) s = 0.5

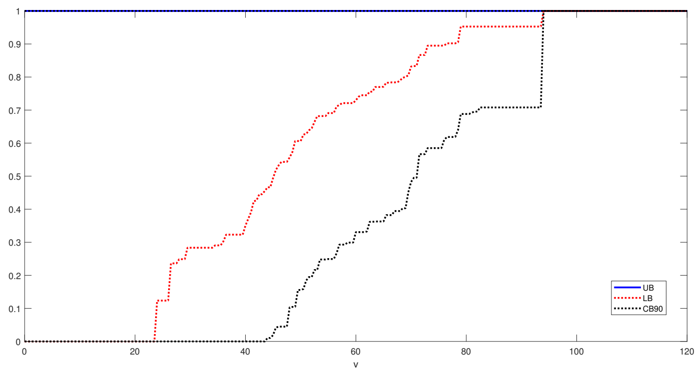


(c) s = 0.65

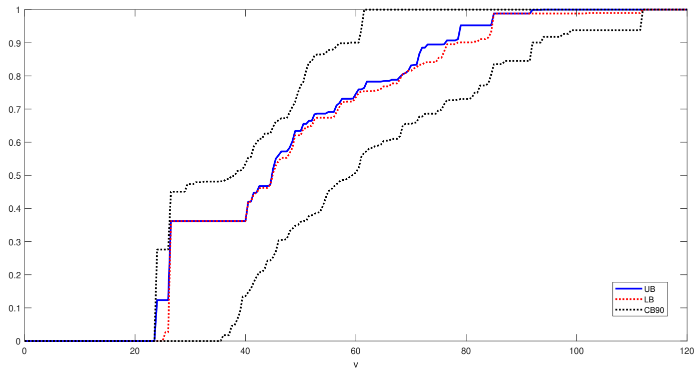
Figure 4: bounds on conditional value CDF – high appraisal. *Note*: we display the upper (resp. lower) bounds in blue solid (resp. red dotted) lines and the 90% confidence band in black dotted lines.

First-price sealed bid auction

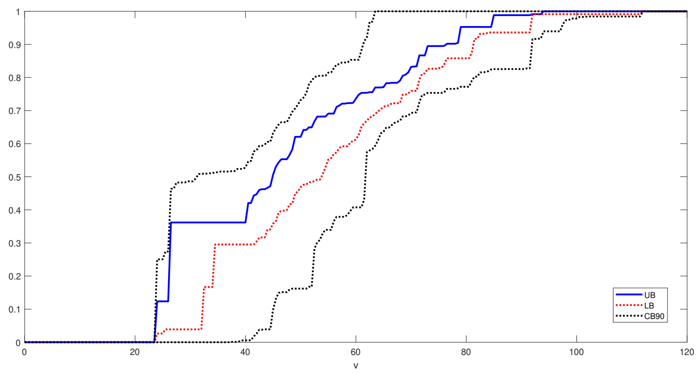
We first conduct a counterfactual analysis by switching from the ascending auction to the first-price sealed bid auction. In each experiment, we first randomly select an auction in the dataset and adopt



(a) s = 0.35



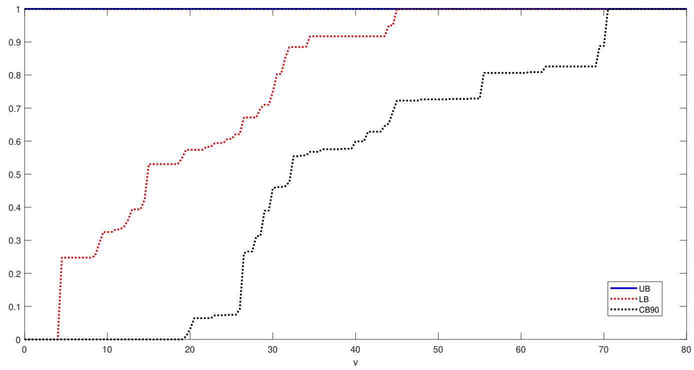
(b) s = 0.5



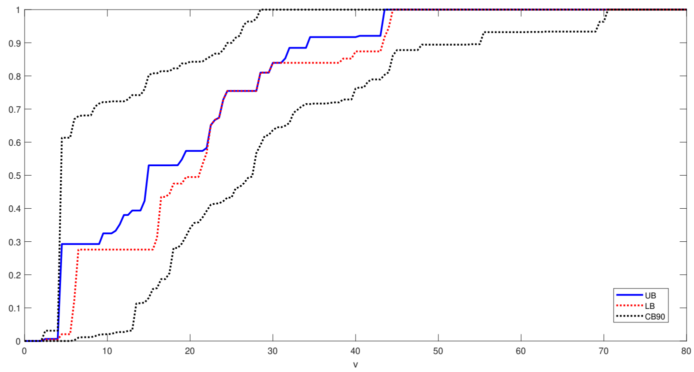
(c) s = 0.65

Figure 5: bounds on conditional value CDF – middle level appraisal. *Note*: we display the upper (resp. lower) bounds in blue solid (resp. red dotted) lines and the 90% confidence band in black dotted lines.

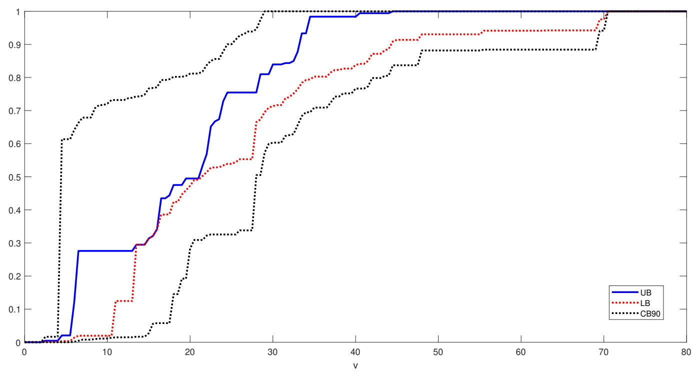
its appraisal value to determine which conditional value distribution bound estimates for simulation. The signal of each bidder is generated from the uniform distribution on $[0, 1]$, and her value is then drawn from the chosen conditional value distribution bound estimates. Second, we calculate the



(a) s = 0.35



(b) s = 0.5



(c) s = 0.65

Figure 6: bounds on conditional value CDF – low appraisal. *Note*: we display the upper (resp. lower) bounds in blue solid (resp. red dotted) lines and the 90% confidence band in black dotted lines.

signal threshold by zero profit condition for entry and determine the bids of all entrants by the bidding strategy of first-price sealed bid auction with entry (see, e.g., Perrigne and Vuong, 2021). We repeat such an experiment for 100,000 times and obtain the 25th, 50th, and 75th percentiles of

Table 5.2: Comparison of revenue – first-price sealed bid auction v.s. ascending auction

revenue percentiles		25%	50%	75%
first-price sealed bid	best scenario (with $\hat{F}^-$ and $\hat{c}^-$)	16.90	39.97	95.75
	worst scenario (with $\hat{F}^+$ and $\hat{c}^+$)	5.08	11.43	25.46
ascending		19.23	44.72	88.10

winning bids.

Table 5.2 reports these results. The best (resp. worst) scenario simulates the values by lower (resp. upper) bound estimate of conditional value distribution and determine the signal threshold by the lower (resp. upper) bound estimate of entry cost. It shows that, in general, ascending auctions yield higher revenue than first-price sealed bid auctions, and the advantage appears mainly in the low and middle percentiles[10]. Specifically, the 25th, 50th percentiles of both the best and worst scenarios are well below the corresponding percentiles of revenue under ascending auction. However, the 75th percentile of revenue of best scenario is a bit higher than the 75th percentile of ascending auction, although the 75th percentile of worst scenario is still well below the ascending auction.

The counterfactual results are consistent with the research of Kagel and Levin (2005). Their main finding is that the winning price of ascending auctions is higher than that of sealed bid auctions because of the disclosure of bidding prices during the bidding process.

Change of entry cost

We now conduct another counterfactual analysis on the revenue change by cutting or adding the entry cost by half. Similar to the first counterfactual analysis, we look at the best (resp. worst) scenario as the one with the lower (resp. upper) bound estimates of conditional value distribution (to generate value given the signal being a random draw from the uniform distribution on $[0, 1]$) and 50% (or 150%) of the lower (resp. upper) bound estimate of entry cost.

Table 5.3 reports the first three quartiles of revenue. It shows that revenue improves when the entry cost is reduced by half. With entry cost cut by half, all revenue quartiles are above those with original entry cost in both the worst and best scenarios. However, it cannot give a clear conclusion in the case of adding the entry cost by half. All quartiles of original revenue are well between the corresponding quartiles of worst and best scenarios.

6 Conclusion

This paper proposes an estimation approach following the constructive identification strategy of Athey and Haile (2002) and Gentry and Li (2014) in the context of ascending auctions with selective entry and establishes the consistency of our estimators in a large sample. Our estimation method is then applied to the alibaba judicial auctions of used cars to recover the bounds of conditional value distribution and the entry cost. Our bounds estimates of conditional value distribution are quite tight for a middle-valued signal. In our counterfactual analysis, we find that (i) the ascending

[10]The revenue equivalence property does not hold here, since the selective entry stage affects the set of active bidders in bidding stage.

Table 5.3: Comparison of revenue – cutting or adding entry cost by half

revenue percentiles		25%	50%	75%
best scenario (with $\hat{F}^{-}$)	50% $\hat{c}^{-}$	62.57	84.07	159.57
	150% $\hat{c}^{-}$	29.40	70.07	127.07
worst scenario (with $\hat{F}^{+}$)	50% $\hat{c}^{+}$	44.57	76.57	144.80
	150% $\hat{c}^{+}$	0	31.38	81.57
actual revenue		19.23	44.72	88.10

auction yields a higher revenue than the first-price sealed bid auction; and (ii) the revenue can be improved significantly when the entry cost is cut by half.

For further research, the treatment of Gentry and Li (2014) in unobserved heterogeneity still applies here. Consider the realization of unobserved heterogeneity $U = u$, by a simple transformation from submitted bids $(B_1, \ldots, B_n)$ to realized bids $(W_1, \ldots, W_N)$, the conditional CDF for W_i is identified:

$$G^*_\omega(b|N;u) = s^*_N(u) + [1 - s^*_N(u)]G^*_b(b|N;u) \tag{9}$$

This is accomplished by applying the identification strategy of Hu et al. (2013). Related articles include Hu (2008) and An et al. (2010).

Furthermore, identification of $s^*_N(u)$ and $G^*_b(b|N;u)$ is implied by identified $G^*_\omega(W_i|N;u)$, which yields bound identification on variables of interest: conditional value distribution $F(v|s,u)$ and entry cost $c(u)$.

Acknowledgment

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Conflict of Intertest

The authors declare no conflict of interest.

Ethics Statement

None declared.

Appendix

A. Proof of Lemma 1

Proof. Part (i) is the last point of Proposition 1 in Gentry and Li (2014). We provide a proof of part (ii) as follows:

$$F^*(v; s_N^*) = \frac{1}{1 - s_N^*} \int_{s_N^*}^{1} F(v|t)dt. \tag{10}$$

Differentiating $F^*(v; s_N^*)$ with respect to s_N^*,we have:

$$\frac{dF^*(v; s_N^*)}{ds_N^*} = \frac{\frac{1}{1-s_N^*} \int_{s_N^*}^{1} F(v|t)dt - F(v|s_N^*)}{1 - s_N^*} \tag{11}$$

By Intermediate Value Theorem, there is a $\tilde{s} \in [s_N^*, 1]$ such that $\frac{1}{1-s_N^*} \int_{s_N^*}^{1} F(v|t)dt = F(v|\tilde{s})$. Under Assumption A (i.2), $\tilde{s} \geq s_N^*$, then $F(v|\tilde{s}) \leq F(v|s_N^*)$. Thus, $\frac{dF^*(v;s_N^*)}{ds_N^*} \leq 0$, and $F^*(v; s_N^*)$ is decreasing with s_N^*. Lemma (ii) is then proved.

B. Proof of Theorem 1

Proof. For any $N \in \{2, \ldots, \overline{N}\}$ and $n = 1, \ldots, N$, $\hat{s}_N$ is consistent by equation (1) and the law of large numbers. In addition, we have

$$\hat{F}^*_{n|N}(v, \hat{s}_N) = \phi_n\big(\hat{G}_{n|N}(v)\big), \tag{12}$$

where $\phi_n(\cdot)$ is defined by $x = n \cdot \phi_n(x)^{n-1} - (n-1) \cdot \phi_n(x)^n$. $\phi_n(\cdot)$ is hence differentiable by the implicit function theorem. The continuous mapping theorem then implies that $\hat{F}^*_{n|N}(v, \hat{s}_N)$ is consistent given that $\hat{G}_{n|N}(v)$ is consistent by the law of large numbers. This further implies that $\hat{F}^*(v, \hat{s}_N)$ is consistent in a large sample.

Given consistent estimators of $\hat{s}_N$ and $\hat{F}^*(v, \hat{s}_N)$, $\hat{F}^+(\cdot|s)$, $\hat{F}^-(\cdot|s)$, $\hat{c}^+$, and $\hat{c}^-$ are all consistent by equations (3)-(6) and the continuous mapping theorem. Therefore, the desired conclusion follows.

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